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http://www3.math.tu-berlin.de/geometrie/Lehre/WS14/DiffGeoII WS 14

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 10

(Geodesics, Gauß' Lemma)

due 15.1.2015

Exercise 1 **5 points**

Let $\mathrm{M} := \{(x, y, z) \mid x^2 + y^2 = 1\} \subset \mathbb{R}^3$ with induced metric. Show that the geodesics on M are curves which have constant angle with the z-axis (i.e. helices, circles and straight lines). Find how many geodesics connect two given points p and q.

Exercise 2 **5 points**

Show that two isometries $F_1, F_2 \colon \mathrm{M} \to \mathrm{M}$ on a Riemannian manifold which agree at a point p and induce the same linear mapping from $\mathrm{T}_p\mathrm{M}$ agree on a neighborhood of p.

Exercise 3 **5 points**

Let $\gamma \colon I \to \mathrm{M}$ be an arc-length parameterized curve. Show that γ is a geodesic if and only if γ is a locally shortest, i.e. for all $t \in I$ exists an $\epsilon > 0$ such that for all $t_1, t_2 \in [t-\epsilon, t+\epsilon]$ the restriction $\gamma|_{[t-\epsilon,t+\epsilon]}$ is the shortest path between $\gamma(t_1)$ and $\gamma(t_2)$. **Hint**: Gauß's lemma.