TECHNISCHE UNIVERSITÄT BERLIN
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Geometry I **WS 2014/15**

http://www3.math.tu-berlin.de/geometrie/Lehre/WS14/GeometryI/

Exercise Sheet 4

$\langle , \rangle$ denotes the Lorentz inner product on $\mathbb{R}^{2,1}$. The last exercise is optional.

Exercise 1. Hyperbolic circles (3 pts)

Define the circle of radius r around point $\mathbf{x_0} \in \mathbf{H}^2$ as

$$C_{\mathbf{x_0},r} := \{\mathbf{x} \subset \mathbf{H}^2 \mid d(\mathbf{x_0}, \mathbf{x}) = r\},$$

where d denotes the hyperbolic distance. What is the length of a hyperbolic circle with radius r? Hint: It is enough to consider circles with center $e_3 = (0,0,1)$. Why?

Exercise 2. Hyperbolic triangle (1.5+2.5 pts)

Given a right-angled hyperbolic triangle ABC.

a) If $\alpha = \frac{\pi}{4}$, $\beta = \frac{\pi}{5}$, and $\gamma = \frac{\pi}{2}$, find the side lengths (a, b, c) of the triangle.

b) Assume $C = (0,0,1)$, and that A is of the form $(\sinh(t), 0, \cosh(t))$ for some $t > 0$. Solve for the coordinates of A and B in $\mathbf{H}^2$.

Exercise 3. Orthogonal lines (2+2 pts)

a) Given a point x and a line l in $\mathbf{H}^2$ not containing x. Show that there exists a unique line m in $\mathbf{H}^2$ containing x and orthogonal to l.

b) Let l_1 and l_2 be hyperbolic lines with normals n_1 and n_2. Show that if $|\langle n_1, n_2\rangle| > 1$, then there exists a unique hyperbolic line l_3 such that $l_1 \perp l_3$ and $l_2 \perp l_3$.

Exercise 4. Parallelism (2.5+2.5 pts)

Let ABC be a right-angled hyperbolic triangle with right-angle at vertex C, with side-lengths (a, b, c) and interior angles $(\alpha, \beta, \frac{\pi}{2})$.

a) Show that $\tanh(a) = \sinh(b)\tan(\alpha)$.

b) Show that $\lim_{a\to\infty} \alpha = \tan^{-1}(\frac{1}{\sinh(b)}) = 2\tan^{-1}(e^{-b})$

Optional Exercise. Half-angle formula (Extra 4 pts)

Adapt the half-angle formula (for $\tan(\frac{\alpha}{2})$) from spherical trigonometry to the hyperbolic plane and prove it.

Due before the first lecture on November 17, 2014