TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Geometry I **WS 2014/15**

http://www3.math.tu-berlin.de/geometrie/Lehre/WS14/GeometryI/

Exercise Sheet 8

Exercise 1. Incidence of Points and Planes (4 pts)

The ambient space for this problem is $\mathbb{R}P^3$.

a) Show that three non-collinear points determine a unique plane containing the three points.

b) Show that three planes not all containing the same line, have a unique point of intersection.

c) Find the plane equation for the plane containing the three points $(1, 1, 0, 1)$, $(0, -1, 1, 1)$, and $(1, 0, 1, 1)$.

d) Show that two lines intersect in a point if and only if they lie in a plane.

Exercise 2. Projective transformations (4 pts)

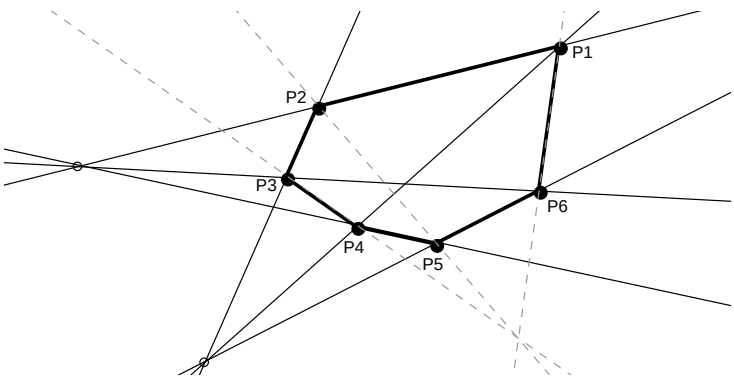
Can a projective transformation $\mathbb{R}\mathrm{P}^1 \to \mathbb{R}\mathrm{P}^1$ which is not the identity have three fixed points? How many fixed points can a projective transformation $\mathbb{R}\mathrm{P}^1 \to \mathbb{R}\mathrm{P}^1$ have? Give an example for each possible case.

Exercise 3. Desargues' theorem in 3D (4 pts)

Let P_1, P_2, P_3, P_4 and Q_1, Q_2, Q_3, Q_4 be two quadruples of pairwise distinct points in $\mathbb{R}P^3$. Then $P_1, .., P_4$ and $Q_1, ..., Q_4$ form two tetrahedra. Suppose that the four lines P_iQ_i intersect in one point. Show that the lines in which planes of corresponding faces intersect (e.g, P_1, P_2, P_3 and Q_1, Q_2, Q_3) lie in a common plane.

Exercise 5. Pappus' theorem (4 pts)

Let $P_1, P_2, P_3, P_4, P_5, P_6$ be distinct points in the projective plane $\mathbb{R}\mathrm{P}^2$. Suppose that the three lines P_1P_2, P_4P_5, P_3P_6 intersect in one point, and the three lines P_2P_3, P_5P_6, P_4P_1 intersect in one point. Show that the lines P_3P_4, P_6P_1, P_5P_2 also intersect in one point. See figure on next page. (Tip: Find Pappus.)



Due before the first lecture on December 15th, 2014