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1. Exercise Sheet – Topology

To be handed in on October 27 after the first lecture.

Homework exercise 1 **5 points**

Let X be a set and let $\mathcal{T} = \{U \subseteq X \,|\, U = \emptyset \text{ or } X \setminus U \text{ is finite}\}$ be the *cofinite topology* on X. Show that $\mathcal{T}$ is indeed a topology on X and that $(X, \mathcal{T})$ is compact. What are all sets X such that X with the cofinite topology is connected?

Homework exercise 2 **5 points**

Let X and Y be topological spaces, $A \subseteq X$, and $B \subseteq Y$. Equip A and B with the subspace topology. Prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

Homework exercise 3 **5 points**

(a) Let for every $n \in \mathbb{N}$ the subset $C_n \subseteq X$ be connected in X with the property that $C_n \cap C_{n+1} \neq \emptyset$ for every $n \in \mathbb{N}$. Show that $\bigcup_{n \in \mathbb{N}} C_n$ is connected.

(b) Let X be a topological space and $x \in X$. If for every $y \in X$ there is a connected set $C \subseteq X$ with $x, y \in C$, then X is connected.

(c) Let $A \subsetneq X$ and $B \subsetneq Y$ be proper subspaces of connected spaces X and Y. Show that $(X \times Y) \setminus (A \times B)$ is also connected.

Homework exercise 4 **5 points**

Show that $\mathbb{R}^d$ and $\mathbb{R}$ with the standard topologies are not homeomorphic if $d > 1$. Construct a topological space X with more than one element such that $X \cong X \times X$.