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## 6. Exercise Sheet – Topology

*To be handed in on December 1 after the first lecture.*

### Homework exercise 1

5 points

Let  $X$  be a path-connected space with path-connected open subsets  $A, B \subseteq X$ . Suppose  $X = A \cup B$ ,  $A$  and  $B$  are not simply connected, but  $A \cap B$  is simply connected. Show that for any  $x_0 \in A \cap B$  and any loop  $\gamma$  at the basepoint  $x_0$  that is not homotopic to a constant loop, there is a loop  $\gamma'$  based at  $x_0$  such that  $\gamma \cdot \gamma' \neq \gamma' \cdot \gamma$ .

### Homework exercise 2

5 points

Let  $X$  be a path-connected Hausdorff space and  $p \in X$  a point with an open neighborhood  $U \cong \mathbb{R}^d$  for some  $d \geq 3$ . Then for any  $x_0 \in X \setminus \{p\}$  we have  $\pi_1(X, x_0) \cong \pi_1(X \setminus \{p\}, x_0)$ .

### Homework exercise 3

5 points

Compute the fundamental group of  $\mathbb{R}P^d$  for every integer  $d \geq 1$ .

### Homework exercise 4

5 points

Compute the fundamental group of  $S^2 \cup \{(x, y, z) \in \mathbb{R}^3 \mid x = y = 0\}$ .