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7. Exercise Sheet – Topology

To be handed in on December 8 after the first lecture.

Homework exercise 1 **5 points**

Let $\gamma_i \colon [0,1] \to \mathbb{R}^3$, $i = 1,2,3$, be the following loops:

$$\gamma_1(t) = (2\cos(2\pi t), \sin(2\pi t), 0),$$
$$\gamma_2(t) = (\cos(2\pi t), 0, 2\sin(2\pi t)), \text{ and}$$
$$\gamma_3(t) = (0, 2\cos(2\pi t), \sin(2\pi t)).$$

The link determined by γ_1, γ_2, and γ_3 is called *Borromean rings*. Show that the Borromean rings are linked by proving that γ_3 is non-trivial in the fundamental group of the complement of γ_1 and γ_2.

Homework exercise 2 **5 points**

Let K be the figure eight knot. Show with the Wirtinger presentation that the fundamental group of the complement has presentation $\pi_1(\mathbb{R}^3 \setminus K) = \langle x, y \mid xyx^{-1}yx = yxy^{-1}xy \rangle$.

Homework exercise 3 **5 points**

A *tricoloring* of a knot diagram is a coloring of the arcs with red, blue, and green such that at every crossing the arcs either all have one color or three distinct colors. A knot diagram is *tricolorable* if a tricoloring with more than one color exists. This is a knot invariant.

Show that the trefoil knot is tricolorable, while the figure eight knot is not tricolorable. Use this to show that the fundamental groups of the complements of these knots are non-isomorphic.

Homework exercise 4 **5 points**

Show that a knot K in $\mathbb{R}^3$ built from four straight segments is trivial. Is the same true for five straight segments?

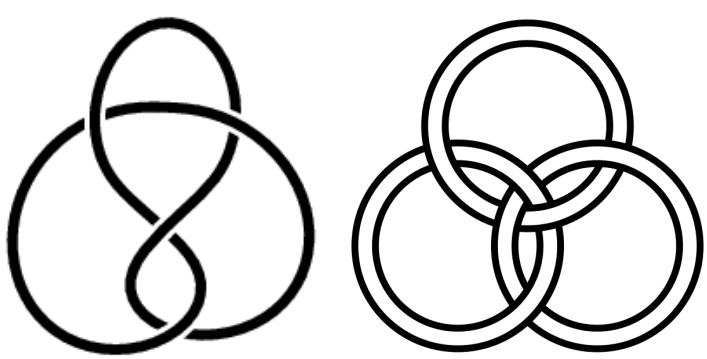

Figure 1: Figure eight knot and Borromean rings