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12. Exercise Sheet – Topology

To be handed in on February 2 after the first lecture.

Homework exercise 1 **5 points**

Let X be an annulus and $A \subseteq X$ its two boundary curves. Let Y be a Möbius strip and $B \subseteq Y$ its boundary curve. Compute the relative homology groups of the pairs (X, A) and (Y, B) and compare them to the homology groups of the quotients X/A and Y/B.

Homework exercise 2 **5 points**

Let G be an abelian group that fits into the short exact sequence $\mathbb{Z}/4 \to G \to \mathbb{Z}/4$. Determine all possible isomorphism types of G. Which of these abelian groups also fit into the short exact sequence $\mathbb{Z}/2 \to G \to \mathbb{Z}/8$?

Hint: Recall the fundamental theorem of finitely generated abelian groups.

Homework exercise 3 **5 points**

Let A be a finite point set on the torus $S^1 \times S^1$. Compute $H_n(S^1 \times S^1, A)$.

Homework exercise 4 **5 points**

Find an example of spaces X and Y and $A \subseteq X$ and $B \subseteq Y$ with $H_i(X) \cong H_i(Y)$ and $H_i(A) \cong H_i(B)$ for all i, but there is an i such that $H_i(X, A) \ncong H_i(Y, B)$.