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13. Exercise Sheet – Topology

To be handed in on February 9 after the first lecture.

Homework exercise 1 **5 points**

Let $A = \{\frac{1}{n} \mid n \in \mathbb{Z}, n > 0\} \cup \{0\}$. Show that $H_1([0,1]/A)$ and $H_1([0,1], A)$ are not isomorphic.

Hint: Hawaiian earrings

Homework exercise 2 **5 points**

Let X be a non-empty space. Compute the homology groups of the cone $\mathrm{C}X$.

Homework exercise 3 **5 points**

Let X be a space. Show that $H_n(X \times S^k) \cong H_n(X) \oplus H_{n-k}(X)$ for all $n \geq k$.

Homework exercise 4 **5 points**

Solve Exercise 2.2.28 in *Hatcher*:

1. Use the Mayer–Vietoris sequence to compute the homology groups of the space obtained from a torus $S^1 \times S^1$ by attaching a Möbius band via a homeomorphism from the boundary circle of the Möbius band to the circle $S^1 \times \{x_0\}$ in the torus.
2. Do the same for the space obtained by attaching a Möbius band to $\mathbb{R}P^2$ via a homeomorphism of its boundary circle to the standard $\mathbb{R}P^1 \subseteq \mathbb{R}P^2$.