Technische Universität Berlin
Institut für Mathematik
Prof. John M. Sullivan
Florian Frick

17.12.2014

Midterm Exam – Topology

Name:

Matrikelnummer:

E-mail (optional):

1	2	3	Σ

You have **80 minutes** to complete this exam. There are three exercises worth 10 points each. The exam is passed successfully with **15 points** or more. You may write your solutions in either German or English. Always show that the prerequisites of theorems you use are satisfied.

Exercise 1 **10 points**

Let

- $X = \mathbb{R}^3 \setminus \{(x, y, z) \in \mathbb{R}^3 \mid x = y = 0\}$, that is, X is $\mathbb{R}^3$ with a line removed,
- let $Y = \mathbb{R}^2 \setminus \{(-1, 0), (1, 0)\}$,
- and let $Z = S^1 \subseteq \mathbb{R}^2$ be the unit circle.

Decide which pairs of spaces X, Y, Z are homotopy equivalent, that is, whether $X \simeq Y$, $Y \simeq Z$, and $X \simeq Z$. Explain your answer.

Exercise 2 **10 points**

Let $u_0 \in S^1$ be fixed. Construct the space X by gluing a disk into the torus $S^1 \times S^1$ along the loop $\{u_0\} \times S^1$ via the map $f\colon S^1 \to S^1 \times S^1, x \mapsto (u_0, x)$. For some $x_0 \in X$ compute $\pi_1(X, x_0)$.

Exercise 3 **10 points**

Let $X = \mathbb{R}P^2 \vee \mathbb{R}P^2$.

(a) Argue that $\pi_1(X) \cong \mathbb{Z}/2 * \mathbb{Z}/2$.

(b) Let $Y = \mathbb{R}P^2 \vee S^2 \vee \mathbb{R}P^2$, where a point of one copy of $\mathbb{R}P^2$ is identified with the north pole of S^2 and a point of the other copy of $\mathbb{R}P^2$ is identified with the south pole. Construct a covering map $p\colon Y \to X$ and determine the subgroup $p_*(\pi_1(Y, y_0))$ of $\pi_1(X, p(y_0))$ for a basepoint $y_0 \in Y$ of your choice.

(c) Construct the universal covering of X geometrically (for instance, as a map from a CW-complex to X).