

## Differential Geometry II: Analysis and Geometry on Manifolds

### Exercise Sheet 1

(Manifolds and examples)

due 26.10.2015

#### Exercise 1

5 points

Let  $X$  be a topological space,  $x \in X$  and  $n \geq 0$ . Show that the following statements are equivalent:

- i) There is a neighborhood of  $x$  which is homeomorphic to  $\mathbb{R}^n$ .
- ii) There is a neighborhood of  $x$  which is homeomorphic to an open subset of  $\mathbb{R}^n$ .

#### Exercise 2

5 points

Let  $n \geq 0$  and let  $J = \{0, \dots, n\}$ . We define  $(U_j^\pm, \varphi_j^\pm)_{j \in J}$  by

$$U_j^\pm := \{x = (x_0, \dots, x_n) \in \mathbb{S}^n \mid \pm x_j > 0\}$$

and

$$\varphi_j^\pm: U_j^\pm \rightarrow \mathbb{R}^n, \quad (x_0, \dots, x_n) \mapsto (x_0, \dots, x_{j-1}, x_{j+1}, \dots, x_n).$$

Show that  $(U_j^\pm, \varphi_j^\pm)_{j \in J}$  is a  $\mathcal{C}^\infty$ -atlas for  $\mathbb{S}^n$ .

#### Exercise 3

5 points

For  $\theta \in \mathbb{R}$  we define  $U_\theta := \mathbb{S}^1 \setminus \{(\cos \theta, \sin \theta)\} \subset \mathbb{S}^1$  and

$$\varphi_\theta: U_\theta \rightarrow (\theta, \theta + 2\pi) \subset \mathbb{R}, \quad (\cos \rho, \sin \rho) \mapsto \rho.$$

- i) Explain why this is well-defined and show that  $(U_\theta, \varphi_\theta)_{\theta \in \mathbb{R}}$  is a  $\mathcal{C}^\infty$ -atlas for  $\mathbb{S}^1$ .
- ii) Show that

$$\{(x_0, x_1) \in \mathbb{S}^1 \mid x_1 > 0\} \rightarrow \mathbb{R}, \quad (x_0, x_1) \mapsto \frac{x_0}{x_1}$$

is another chart for  $\mathbb{S}^1$  and decide whether it is in the  $\mathcal{C}^\infty$ -structure defined by the atlas  $(U_\theta, \varphi_\theta)_{\theta \in \mathbb{R}}$ .