Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 2

(Manifolds and smooth maps)

due 2.11.2015

Exercise 1 **5 points**

Let M be a topological manifold. Show that M has an *exhaustion by compact sets*, i.e. there exists a sequence $K_1 \subset K_2 \subset K_2 \subset \cdots \subset M$ of compact sets such that $K_i \subset \mathring{K}_{i+1}$ (the interior of K_{i+1}) and $\bigcup_{i=1}^{\infty} K_i = M$.

Exercise 2 **5 points**

Let M and N be smooth manifolds of dimension m and n, respectively. Show that the product $M \times N$ is a smooth manifold of dimension $m + n$. Is this also true for smooth manifolds with boundary?

Exercise 3 **5 points**

Let M_1, M_2 and N be smooth manifolds and $f\colon N \to M_1 \times M_2$ be a map. For $i \in \{1, 2\}$ we define on the projection on the i-th component:

$$\pi_i\colon M_1 \times M_2 \to M_i, \quad \pi_i(x_1, x_2) := x_i.$$

Prove that f is smooth if and only if $\pi_1 \circ f$ and $\pi_2 \circ f$ are smooth.