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http://www3.math.tu-berlin.de/geometrie/Lehre/WS15/DGII/ WS 15

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 3

(Tangent bundles, vector fields, submanifolds)

due 9.11.2015

Exercise 1 **5 points**

Prove that the tangent bundle of a product of smooth manifolds is diffeomorphic to the product of the tangent bundles of the manifolds. Deduce that the tangent bundle of a torus $\mathbb{S}^1 \times \mathbb{S}^1$ is diffeomorphic to $\mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{R}^2$.

Exercise 2 **5 points**

Show that each of the following conditions is equivalent to the smoothness of a vector field X as a section $X\colon M \to TM$:

a) For each $f \in \mathscr{C}^\infty(M)$, the function Xf is also smooth.
b) If we write $X|_U =: \sum v_i \partial_i$ in a coordinate chart (U, φ), then the components $v_i\colon U \to \mathbb{R}$ are smooth.

Exercise 3 **5 points**

Let $\mathrm{Sym}(3) \cong \mathbb{R}^6$ denote the set of real symmetric 3×3 matrices. Let

$$M := \{P \in \mathrm{Sym}(3) \mid P^2 = P,\ tr(P) = 1\}.$$

Show that $M \subset \mathrm{Sym}(3)$ is a submanifold diffeomorphic to $\mathbb{RP}^2$.