TECHNISCHE UNIVERSITÄT BERLIN
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Geometry I - WS 15

http://www.math.tu-berlin.de/geometrie/Lehre/WS15/GeometryI/

Exercise Sheet 1

Exercise 1: Distance. (2 pts)

Let x and y be two points in Euclidean space $\mathbb{R}^n$. Prove that the shortest path between them is the line segment connecting them.

Exercise 2: Double angles. (6 pts)

Let $A \neq B$ be two points on a circle centered at a point M in the plane. These points split the circle into two arcs. Let $\widehat{AB}$ denote the longer arc (or either arc, if they are equal semicircles). Prove that for any point $P \in \widehat{AB}$, the angles $\angle AMB$ and $\angle APB$ satisfy the relation

$$\angle AMB = 2\,\angle APB.$$

What relation holds if P is in the smaller arc? If the line segment $\overline{AB}$ is a diameter, deduce that $\angle APB = \pi/2$ for any point $P \neq A, B$ on the circle.

Using this (or otherwise) prove that for a convex quadrilateral $ABCD$ inscribed in a circle, the two pairs of opposite angles satisfy the relation

$$\angle DAB + \angle BCD = \pi = \angle ABC + \angle CDA.$$

Exercise 3: Spherical circles. (4 pts)

In $\mathbb{S}^2$, define the circle of radius r around a point c as

$$C_r(c) := \left\{x \in \mathbb{S}^2 \mid d(c,x) = r\right\},$$

where $d(\cdot,\cdot)$ is the spherical metric. Show that $C_r(c)$ is the intersection of $\mathbb{S}^2$ with a plane. Find the circumference of the circle.

Exercise 4: Perpendicular bisector. (4 pts)

Given two points $P \neq Q \in \mathbb{S}^2$, define

$$X := \left\{x \in \mathbb{S}^2 \mid d(x,P) = d(x,Q)\right\}.$$

Show that X is a great circle that intersects any great circle through P and Q orthogonally. What is special about the case $P = -Q$?

Due: Before lecture on Tuesday, 27.10.2015.