

Exercise Sheet 6

Exercise 1: Cross-ratio of four points on a conic. (4 pts)

Given a non-degenerate conic $\mathcal{C} \subset \mathbb{RP}^2$. The cross-ratio of four points A, B, C , and D on \mathcal{C} is defined by $cr(A, B, C, D) = cr(PA, PB, PC, PD)$, where P is another arbitrary point on \mathcal{C} .

Let $\mathcal{C} \subset \mathbb{RP}^2$ be a non-degenerate conic. Let P, Q , and R be such that \mathcal{C} is tangent to PQ at Q and PR at R . Prove that for any $A, B \in \mathcal{C}$ the following formula holds:

$$(cr(Q, A, R, B))^2 = cr(PQ, PA, PR, PB).$$

Exercise 2: Pencil of conics with four common points. (4 pts)

Consider the one-parameter family of conics through the four points $[1, 1, 1], [1, -1, 1], [-1, -1, 1], [-1, 1, 1] \in \mathbb{RP}^2$.

- (i) Find its general formula.
- (ii) Show that any two conics of this family have only the given four points in common.
- (iii) What is the equation of the unique conic in this family that passes through the point $P = [2, 0, 1]$?
- (iv) Draw a picture of this family.

Exercise 3: Pencil of conics with two common points. (4 pts)

Construct a pencil of conics in \mathbb{RP}^2 with two common points, i.e. a line in the space of conics such that each two conics from the family intersect in exactly the same two points. Draw a picture of this family

Exercise 4: Tangents of a conic. (4 pts)

Let \mathcal{C} be a conic in \mathbb{RP}^2 and P a point on \mathcal{C} . Give a geometric construction for the tangent of \mathcal{C} at P only using a ruler (one can draw lines, but cannot measure distances).