

DIFFERENTIAL GEOMETRY II: ANALYSIS AND GEOMETRY ON MANIFOLDS

Exercise Sheet 10

(Geodesics, exponential map)

due 15.1.2018

Exercise 1

5 points

- a) Is there a Riemannian manifold (M, g) which has finite diameter (i.e. there is an m such that all points $p, q \in M$ have distance $d(p, q) < m$) and there is a geodesic of infinite length without self-intersections?
- b) Find an example for a Riemannian manifold diffeomorphic to \mathbb{R}^n but which has no geodesic of infinite length.

Exercise 2

5 points

Let (M, g) be a Riemannian manifold of dimension n . Show that at each point $p \in M$ there is a local coordinate $\varphi = (x_1, \dots, x_n)$ at p such that

$$g\left(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right)\Big|_p = \delta_{ij}, \quad \nabla \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \Big|_p = 0.$$

Exercise 3

5 points

Show that two isometries $F_1, F_2: M \rightarrow M$ which agree at a point p and induce the same linear mapping from $T_p M$ agree on a neighborhood of p .