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http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/DGII/ WS 17

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 11

(Geodesics, Hopf-Rinow theorem)

due 22.01.2017

Exercise 1 **5 points**

Let M be a Riemannian manifold. A curve $\gamma\colon [0,a) \to M$ is called *divergent*, if for every compact set $K \subset M$ there exists a $t_0 \in [0,a)$ such that $\gamma(t) \notin K$ for all $t > t_0$. Show: M is complete if and only if all divergent curves are of infinite length.

Exercise 2 **5 points**

Let M be a compact Riemannian manifold. Show that M has finite diameter, and that any two points $p, q \in M$ can be joined by a geodesic of length $d(p,q)$.

Exercise 3 **5 points**

Let M be a complete Riemannian manifold, which is not compact. Show that there exists a geodesic $\gamma\colon [0,\infty) \to M$ which for every $s > 0$ is the shortest path between $\gamma(0)$ and $\gamma(s)$.