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http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/DGII/ WS 17

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 12

(Jacobi fields)

due 29.01.2018

Exercise 1 **5 points**

Let (M, g) be a Riemannian manifold with Levi-Civita connection ∇, and Y be a *Killing field*, i.e. a vector field $Y \in \Gamma(TM)$ such that ∇Y is skew-adjoint:

$$g(\nabla_X Y, Z) = -g(X, \nabla_Z Y)$$

for all $X, Z \in \Gamma(TM)$. Further, let γ be a geodesic. Show:

$$g(Y, \gamma') \textit{ is constant.}$$

Exercise 2 **5 points**

Let M be a Riemannian manifold, $p \in M$ and $V, W \in T_pM$ such that $\exp(V)$ is defined. Further, let Y be the Jacobi field along $\gamma(t) = \exp(tV)$ given by the initial conditions $Y(0) = 0,\ Y'(0) = W$. Show:

$$d_{tV} \exp(tW) = Y(t) \ for\ t \in [0, 1].$$

Exercise 3 **5 points**

a) Show that the isometries of $\mathbb{S}^n$ are restrictions of isometries of $\mathbb{R}^{n+1}$ which map $\mathbb{S}^n$ to itself.
b) Find the Jacobi fields of $\gamma_1 \colon \mathbb{R} \to \mathbb{R}^n,\ t \mapsto p + tX$, where $p, X \in \mathbb{R}^n$.
c) Find the Jacobi fields of $\gamma_2 \colon \mathbb{R} \to \mathbb{S}^n \subset \mathbb{R}^{n+1},\ t \mapsto \cos(t)p + \sin(t)X$, where $p \in \mathbb{S}^n, X \in T_p\mathbb{S}^n,\ |X| = 1$.