Technische Universität Berlin
Institut für Mathematik

Berlin Mathematical School

Mathematical Physics I, Dr. Matteo Petrera, Mats Vermeeren WS 17/18

http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/MP1/

Exercise Sheet 1 (19.10.17)

Due date: ***26.10.17***

- To get the Übungsschein (necessary condition for the oral exam) you need to obtain an *average of 60% in each half of the semester*, i.e. on the first 7 sheets and on the remaining 6 or 7 sheets. Each exercise sheet has 25 points.
- Please work in fixed groups of *2 students*.
- Justify each step of your computations. *Results without any explanation are not accepted.* Please write in a readable way. *Unreadable handwriting will not be corrected.* You can write your answers in English or in German.
- Please turn in your homework at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed.*

Exercise 1 (2+2+1 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = |x|^{p/q}, \\ x(0) = 0, \end{cases}$$

with $p, q \in \mathbb{N} \setminus \{0\}$.

1. Prove that it has a unique solution if $p > q$.
2. Prove that it has an infinite number of solutions if $p < q$.
3. What can you say if $p = q$?

Exercise 2 (5 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = \dfrac{x^2}{x^2 + \epsilon} \sqrt{|x|}, \\ x(0) = 0, \end{cases}$$

with $\epsilon > 0$. What can you say about existence and uniqueness of its solutions? Is the solution unique if $\epsilon = 0$?

Exercise 3 (5 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = t + x, \\ x(0) = 1. \end{cases}$$

Construct the sequence of Picard iterations and obtain the explicit solution.

Turn over

Exercise 4 (2+3 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = x^3 - x, \\ x(0) = \dfrac{1}{2}. \end{cases}$$

1. What can you say about existence and uniqueness of its solutions?
2. Without solving the ODE, calculate $\lim_{t\to+\infty} \phi(t)$, where $\phi(t)$ is the solution.

Exercise 5 (4+1 pts)

Consider the parameter-dependent map

$$x_{n+1} = \Phi(x_n, \alpha) := \alpha x_n(1 - x_n).$$

For $\alpha \in [0, 4]$, Φ maps the interval $[0, 1]$ to itself.

Implement an iteration of this map in your favorite programming language or mathematical software. For example, in Sage[1] your code might look like:

```
x = [initial_value]
for i in [1..number_of_iterates]:
    x += [alpha*x[i-1]*(1-x[i-1])]
list_plot(x)
```

It is not necessary to submit your code.

1. Fix an initial value $x_0 \in [0, 1]$ and run the iteration for a sufficiently long time to judge the limit behavior. Do this for various values of the parameter $\alpha \in [0, 4]$.

   How does the behavior change? Try to find the precise values of α where changes happen.
2. Prove your observations for $\alpha < 1$.

[1] Open source, available at `http://www.sagemath.org/`