Technische Universität Berlin
Institut für Mathematik

Mathematical Physics I, Dr. Matteo Petrera, Mats Vermeeren WS 17/18
http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/MP1/

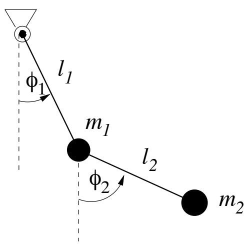
Exercise Sheet 11 (25.01.18)

Due date: ***01.02.18***

- To get the Übungsschein (necessary condition for the oral exam) you need to obtain an *average of 60% in each half of the semester*, i.e. on the first 7 sheets and on the remaining 6 or 7 sheets. Each exercise sheet has 25 points.
- Please work in fixed groups of *2 students.*
- Justify each step of your computations. *Results without any explanation are not accepted.* Please write in a readable way. *Unreadable handwriting will not be corrected.* You can write your answers in English or in German.
- Please turn in your homework at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed.*

Exercise 1 (1+4+3 pts)

Consider the mechnical system in the figure below. The rods (lengths ℓ_1, ℓ_2) connecting the masses m_1 and m_2 are rigid and massless, and the whole system is frictionless. Assume a constant vertical gravitational field.



1. What is the configuration manifold of the system?
2. Write down the Lagrangian of the system.
3. Write down the Euler-Lagrange equations.

Exercise 2 (4 pts)

Determine the equations of motion a free particle (point mass) on the torus: consider a Lagrangian equal to the kinetic energy, restrict it to the Torus in suitable variables, and write down the resulting Euler-Lagrange equations.

(Denote by R be the radius from the origin in $\mathbb{R}^3$ to the center of the Torus tube, and by r the radius of the tube.)

Turn over

Exercise 3 (1+3+2 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^2$ consider a point of mass $m > 0$ with Hamiltonian

$$\mathcal{H}(q,p) := \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2, \qquad \omega > 0.$$

Introduce the complex variables

$$a_\pm(q,p) := \sqrt{\frac{m\omega}{2}}\left(q \pm \frac{ip}{m\omega}\right), \qquad i^2 = -1.$$

1. Write the Hamiltonian in terms of $a_\pm$.
2. Compute the Poisson brackets $\{a_+, a_-\}$ and $\{a_\pm, \mathcal{H}\}$.
3. Find and solve the equations of motion for the variables $a_\pm$. Determine the general real solution of the equations of motion in terms of the variables q and p.

Exercise 4 (4+3 pts)

Consider two unit mass point-like particles in $\mathbb{R}^3$. The first particle moves along a circle, in a horizontal plane, parametrized by

$$(x_1, x_2, 0) = (r\cos\theta, r\sin\theta, 0), \qquad \theta \in \mathbb{R}/2\pi\mathbb{Z};$$

the second particle is constrained on the curve parametrized by

$$(y_1, y_2, y_3) = (r\cos\phi, r\sin\phi, h\sin\phi), \qquad \phi \in \mathbb{R}/2\pi\mathbb{Z},$$

where $r > 0$ and $h > 0$ are fixed. Assume that the two particles attract each other with a force of magnitude $F = kd$, where d is the distance between the particles. There are no external forces

1. Write down the Lagrangian of the system.
2. Write down the Euler-Lagrange equations.