Technische Universität Berlin
Institut für Mathematik

Berlin Mathematical School

Mathematical Physics I, Dr. Matteo Petrera, Mats Vermeeren WS 17/18

http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/MP1/

Exercise Sheet 12 (01.02.18)

Due date: ***08.02.18***

- To get the Übungsschein (necessary condition for the oral exam) you need to obtain an *average of 60% in each half of the semester*, i.e. on the first 7 sheets and on the remaining 6 or 7 sheets. Each exercise sheet has 25 points.
- Please work in fixed groups of *2 students.*
- Justify each step of your computations. *Results without any explanation are not accepted.* Please write in a readable way. *Unreadable handwriting will not be corrected.* You can write your answers in English or in German.
- Please turn in your homework at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed.*

Exercise 1 (2+4+2 pts)

Consider the planar system of ODEs

$$\begin{cases} \dot{x} = x\left(1 - \frac{x}{2} - y\right), \\ \dot{y} = y\left(x - 1 - \frac{y}{2}\right), \end{cases}$$

1. Determine all fixed points.
2. Linearize around each of the fixed points. Determine their type.
3. Sketch the phase portrait.

Exercise 2 (2+3+2 pts)

In $\mathbb{R}^2$, consider the Hamiltonian

$$H(q,p) = -\frac{1}{3}\left(p + \frac{1}{2}\right)\left(p + \sqrt{3}q - 1\right)\left(p - \sqrt{3}q - 1\right).$$

1. Write down the canonical Hamilton equations.
2. Find all fixed points. Determine their stability.
3. Sketch the phase portrait.

Turn over

Exercise 3 (3+1 pts)

Consider the scalar ODE
$$\dot{x} = \alpha x - \sin x, \qquad \alpha \in \mathbb{R}.$$

1. Study the local bifurcations that occur near $x = 0$ as α is varied. Draw the bifurcation diagram.

2. Which type of local bifurcation occurs at many points other than $x = 0$?

Exercise 4 (3+3 pts)

In $\mathbb{R}^2$, consider the Lagrangian
$$\mathcal{L}(q, \dot{q}) = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2\right) + 2\alpha^2(q_1\dot{q}_2 - q_2\dot{q}_1)^2 - \alpha\beta(q_1^2 + q_2^2), \qquad \alpha, \beta > 0.$$

1. Use the Noether Theorem to find an integral of motion.

2. Determine the value of this integral for which the circle $q_1^2 + q_2^2 = c$, $c > 0$, is an orbit.