Technische Universität Berlin
Institut für Mathematik

Mathematical Physics I, Dr. Matteo Petrera, Mats Vermeeren — WS 17/18

http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/MP1/

Bonus Exercise Sheet (08.02.18)

Due date: ***15.02.18***

- To get the Übungsschein (necessary condition for the oral exam) you need to obtain an *average of 60% in each half of the semester*, i.e. on the first 7 sheets and on the remaining 6 or 7 sheets. Each exercise sheet has 25 points.
- Please work in fixed groups of *2 students*.
- Justify each step of your computations. *Results without any explanation are not accepted.* Please write in a readable way. *Unreadable handwriting will not be corrected.* You can write your answers in English or in German.
- Please turn in your homework at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed.*

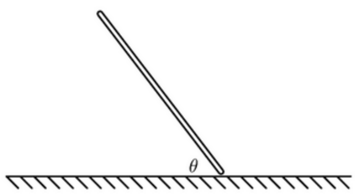
Exercise 1 (5 pts)

Consider a conservative force field $F(x) = -\nabla U(x)$ in $\mathbb{R}^2$ whose equipotential lines ($\{U = \text{const}\}$) are straight. Consider a particle of mass m in this field. Prove that along any trajectory of such a particle, one has $|\dot{x}| \sin\theta = \text{const}$, where θ is the angle between the normal to the equipotential and the trajectory.

Bonus question. Does this still hold if the equipotential lines are cruved?

Exercise 2 (5 pts)

Consider a uniform rod falling on a horizontal plane of ice. One endpoint of the rod touches the ice from the start, and can move along it without friction.



Find a differential equation describing the time evolution of the angle θ that the rod makes with the plane, that is valid as long as the endpoint keeps contact with the plane.

Bonus question. At which angle will the rod lose contact with the plane?

Turn over

Exercise 3 (5 pts)

Consider a Hookean spring with rest length 0 (i.e. its potential energy at length ℓ is $\frac{1}{2}k\ell^2$) wrapped around the cylinder $x^2 + y^2 = 1$ in $\mathbb{R}^3$. Assume that there is no friction between the cylinder and the spring, and that the spring is at rest with its endpoints at $(1, 0, 0)$ and $(1, 0, h)$ for some $h > 0$. Give all possible shapes of the spring.

Bonus question. Find a system involving a moving point mass that obeys exactly the same equations.