TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Mathematical Physics I, Dr. Matteo Petrera, Mats Vermeeren WS 17/18

http://www3.math.tu-berlin.de/geometrie/Lehre/WS17/MP1/

Exercise Sheet 9 (11.01.18)

Due date: ***18.01.18***

- To get the Übungsschein (necessary condition for the oral exam) you need to obtain an *average of 60% in each half of the semester*, i.e. on the first 7 sheets and on the remaining 6 or 7 sheets. Each exercise sheet has 25 points.
- Please work in fixed groups of *2 students*.
- Justify each step of your computations. *Results without any explanation are not accepted.* Please write in a readable way. *Unreadable handwriting will not be corrected.* You can write your answers in English or in German.
- Please turn in your homework at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed.*

Exercise 1 (4 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^{2n}$ consider the transformation

$$(q,p) \mapsto (\widetilde{q},\widetilde{p}) := (q, f(q,p)),$$

for some smooth function f. Determine the structure that f must have for the transformation to be symplectic.

Exercise 2 (4 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^2$ consider the transformation

$$(q,p) \mapsto (\widetilde{q},\widetilde{p}) := \left(q\sqrt{1+q^2\,p^2}, \frac{p}{\sqrt{1+q^2\,p^2}} \right).$$

Show that this transformation is symplectic.

Exercise 3 (4+1+3 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^4$ consider the Hamiltonian

$$\mathcal{H}(q_1,q_2,p_1,p_2) := \frac{1}{2}\left(p_1^2 + q_1^2\,q_2\,p_2\right).$$

1. Find a one-parameter group of symplectic transformations

$$(q_1,q_2,p_1,p_2) \mapsto (\widetilde{q}_1,\widetilde{q}_2,\widetilde{p}_1,\widetilde{p}_2) = \Psi_s(q_1,q_2,p_1,p_2), \qquad s\in\mathbb{R},$$

which preserves the form of the function $\mathcal{H}$ for all $s\in\mathbb{R}$, i.e.,

$$\mathcal{H}(\widetilde{q}_1,\widetilde{q}_2,\widetilde{p}_1,\widetilde{p}_2) = \frac{1}{2}\left(p_1^2 + q_1^2\,q_2\,p_2\right).$$

Hint: split the variables in pairs (q_1,p_1) and (q_2,p_2). You can choose transformations that act as the identity on one of these pairs.

Turn over

2. Construct the infinitesimal generator $\mathbf{v}$ of Ψ_s.

3. Find a Hamiltonian of the vector field $\mathbf{v}$ and verify that this new Hamiltonian is an integral of motion of the flow of the original Hamiltonian $\mathcal{H}$.

Exercise 4 (3+3+3 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^2$ consider the the parametric family of vector fields

$$f(q,p) := \left(p^\alpha q^\beta, -p^{\alpha+1} q^\delta\right),$$

with $\alpha, \beta, \delta \in \mathbb{R}$.

1. Find the values of α, β, δ for which the f is Hamiltonian.

2. Compute the corresponding Hamiltonians.

3. Solve the canonical Hamilton equations for $\alpha \neq -1$.