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http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/DGII/ WS 18

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 3

(Immersions, vector fields)

due 13.11.2018

Exercise 1 **5 points**

Let $f\colon S \to M$ be a *smooth immersion*, i.e. for every $p \in S$ the differential $d_pf\colon T_pS \to T_{f(p)}M$ is injective. Show that for each $p \in S$ there is an open set $U \subset S$ with $U \ni p$ such that $f(U)$ is a submanifold of M.

Exercise 2 **5 points**

Let $f\colon S \to M$ be a smooth immersion. Prove: If f is moreover a *topological embedding*, i.e. its restriction $f\colon S \to f(S)$ is a homeomorphism between S and $f(S) \subset M$ (with its subspace topology), then $f(S)$ is a smooth submanifold of M.

Exercise 3 **5 points**

Show that each of the following conditions is equivalent to the smoothness of a vector field X as a section $X\colon M \to \mathrm{T}M$:

a) For each $f \in \mathscr{C}^\infty(M)$, the function Xf is also smooth.
b) If we write $X|_U =: \sum v_i \frac{\partial}{\partial x_i}$ in a coordinate chart $\varphi = (x_1, \ldots, x_n)$ defined on $U \subset M$, then the components $v_i\colon U \to \mathbb{R}$ are smooth.