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http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/DGII/ WS 18

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 9

(Geodesics, exponential map, isometries)

due 08.01.2017

Exercise 1 **5 points**

Let (M, g) be a Riemannian manifold of dimension n. Show that at each point $p \in M$ there is a local coordinate $\varphi = (x_1, \ldots, x_n)$ at p such that

$$g\big(\tfrac{\partial}{\partial x_i}, \tfrac{\partial}{\partial x_j}\big)\Big|_p = \delta_{ij}, \qquad \nabla_{\frac{\partial}{\partial x_i}} \frac{\partial}{\partial x_j}\Big|_p = 0.$$

Exercise 2 **5 points**

Show that two isometries $f_1, f_2 \colon M \to M$ which agree at a point p and induce the same linear mapping from T_pM agree on a neighborhood of p.

Exercise 3 **5 points**

Let M be a Riemannian manifold, $\gamma\colon I \to M$ be a curve which is parametrized with constant speed, and $f\colon M \to M$ be an isometry which fixes γ, i.e. $f \circ \gamma = \gamma$. Furthermore, let

$$\ker(\mathrm{id} - d_{\gamma(t)} f) = \mathbb{R}\gamma'(t), \ \textit{for all } t.$$

Then γ is a geodesic.