TECHNISCHE UNIVERSITÄT BERLIN
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Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 1

Exercise 1: Distance. (2 pts)

Let x and y be two points in the Euclidean space $\mathbb{R}^n$. Prove that the shortest continuously differentiable curve between them is the line segment connecting them.

Exercise 2: Double angles. (6 pts)

(a) Prove that opposite angles in a convex quadrilateral inscribed in a circle sum to π.

(b) Consider a convex quadrilateral with edges tangent to a circle. Prove that the sum of edge lengths of opposite edges is equal, that is, if the edge lengths are a, b, c, d, then

$$a + c = b + d.$$

Exercise 3: Spherical circles. (4 pts)

In S^2, define the circle of radius r around a point c as

$$C_r(c) := \left\{ x \in S^2 \mid d(c, x) = r \right\},$$

where $d(\cdot, \cdot)$ is the spherical metric. Show that $C_r(c)$ is the intersection of S^2 with a plane. Find the perimeter of the circle.

Exercise 4: Perpendicular bisector. (4 pts)

Given two points $P \neq Q \in S^2$, define

$$X := \left\{ x \in S^2 \mid d(x, P) = d(x, Q) \right\}.$$

Show that X is a great circle that intersects any great circle through P and Q orthogonally. What is special about the case $P = -Q$?

Due: Before lecture on Wednesday, 31.10.2018.