TECHNISCHE UNIVERSITÄT BERLIN
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Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 10

Exercise 1: Cross ratio. (3 pts)

Find all $t \in \mathbb{C}$ such that there exists a projective transformation $\tau : P^1(\mathbb{C}) \to P^1(\mathbb{C})$ with $\tau(0) = 0, \tau(1) = 1, \tau(t) = 2, \tau(2) = 6 - t$.

Exercise 2: Pappus. (4 pts)

Let $P_1, P_2, P_3, P_4, P_5, P_6$ be distinct points in the projective plane $\mathbb{RP}^2$. Suppose that the three lines P_1P_2, P_4P_5, P_3P_6, as well as the three lines P_2P_3, P_5P_6, P_4P_1 intersect at one point. Show that the lines P_3P_4, P_6P_1, P_5P_2 also intersect at one point.

Tip: Make a sketch and find Pappus.

Exercise 3: Cross ratio in space. (4 pts)

Let l_1, l_2, l_3 be three pairwise skew lines in $\mathbb{RP}^3$ and a, b, c, d four distinct lines in $\mathbb{RP}^3$ which intersect each line l_i. Then the four intersection points $\{a_i, b_i, c_i, d_i\}$ on l_i determine a cross ratio $q_i = cr(a_i, b_i, c_i, d_i)$. Show that $q_1 = q_2 = q_3$.

Exercise 4: Cross ratio again. (5 pts)

Let E_1, E_2, E_3, E_4 be four distinct planes in $\mathbb{RP}^3$ that all contain line l. Show that the quantities c_1 and c_2 defined below are equal, that is, $c_1 = c_2$.

(a) Let E' be a plane that does not contain the line l. Let l_i be the line of intersection of E_i and E'. Let $c_1 = cr(l_1, l_2, l_3, l_4)$.

(b) Let l' be a line skew to l and let P_i be the intersection of l' with E_i. Let $c_2 = cr(P_1, P_2, P_3, P_4)$.

Due: Before lecture on Wednesday, 16.1.2019.