TECHNISCHE UNIVERSITÄT BERLIN
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Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 11

Exercise 1. Projectivities between lines in $\mathbb{R}\mathrm{P}^3$ (4 pts)

Define $A_0 = [(1,0,0,0)], B_0 = [(0,1,0,0)], A_1 = [(0,0,1,0)], B_1 = [(0,0,0,1)] \in \mathbb{R}\mathrm{P}^3$. Let l_0 be the line through A_0 and B_0 and l_1 be the line through A_1 and B_1. Let l_2 be an arbitrary line skew to both l_0 and l_1, and let A_2 and B_2 be two points of l_2, so that $P = [sA_2 + tB_2]$ is a general point of l_2. Let Π be the plane spanned by l_0 and P and define Q to be the intersection of Π with l_1. Let f be the map $f : l_2 \to l_1$ defined by $f(P) = Q$. Find a matrix corresponding to f.

Exercise 2. Involutions in $\mathbb{R}\mathrm{P}^1$ (3 pts)

Let X be a set. An involution of X is a map $f : X \to X$ such that $f \circ f = id$. Given a projective transformation f of $\mathbb{R}\mathrm{P}^1$ show that f can be factored as the product (concatenation) of three involutions.

Exercise 3. Image analysis (4 pts)

The book in the photo on the opposite page measures 16cm x 24cm. What are the dimensions of the table? Show your work.

Hint: Construct vanishing points and use cross ratios.

Exercise 4. Projective reflections of $\mathbb{R}\mathrm{P}^n$ (5 pts)

In the following exercise, we denote a representative vector of point $P \in \mathbb{R}\mathrm{P}^n$ by small letter p. For $x, y \in \mathbb{R}^{n+1}$ we denote by $\langle x, y\rangle$ the Euclidean scalar product on $\mathbb{R}^{n+1}$.

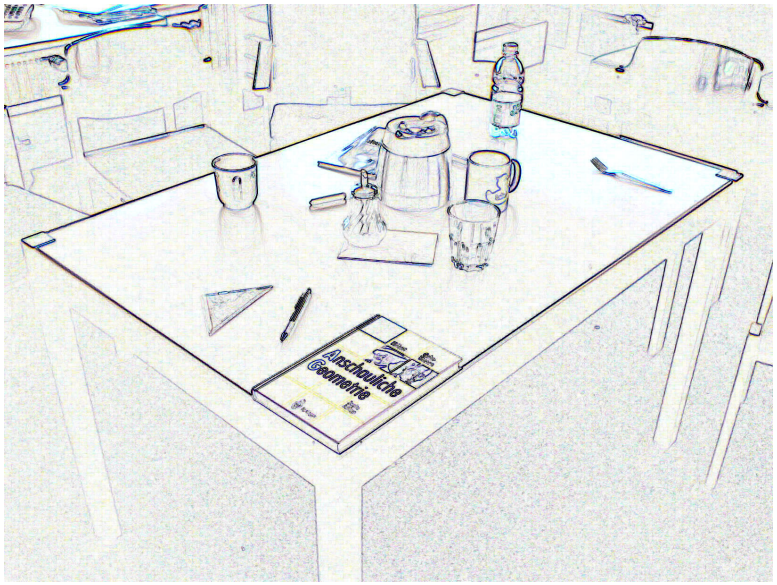
Let $P \in \mathbb{R}\mathrm{P}^n$ and Π a hyperplane in $\mathbb{R}\mathrm{P}^n$ given by $\Pi = \{V \in \mathbb{R}\mathrm{P}^n \mid \langle v, n\rangle = 0\}$ for some fixed $N \in \mathbb{R}\mathrm{P}^n$. Then the *projective reflection in the hyperplane* Π *with center* P is the projective transformation $f : \mathbb{R}\mathrm{P}^n \to \mathbb{R}\mathrm{P}^n$ with fixed point set $F = \{P\} \cup \Pi$, whose action on $X \notin F$, is defined as follows: construct the line l joining P and X, and find its unique intersection Q with Π. Then $f(X)$ lies on l and satisfies $\mathrm{cr}(f(X), Q, X, P) = -1$.

(a) Show that f is well-defined.

(b) Show that $f([x]) = \left[x - 2\frac{\langle x,n\rangle}{\langle p,n\rangle}p\right]$.

(c) Calculate the element of $PGL(3,\mathbb{R})$ for the harmonic reflection of $\mathbb{R}\mathrm{P}^2$ with center $P = [(1,1,1)]$ and Π is equal to the line $u_1 + u_2 = 1$ in affine coordinates $u_1 = \frac{x_1}{x_3}, u_2 = \frac{x_2}{x_3}$, where x_1, x_2, x_3 are homogeneous coordinates.

Due: Before lecture on Wednesday, 23.1.2019.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