TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. Boris Springborn
Hana Kouřimská

Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 12

Exercise 1: Annihilator space. (4 pts)

Let U_1 and U_2 be two subspaces of a finite-dimensional vector space V. Prove the following identities:

(a) $(U_1 \cap U_2)^o = U_1^o + U_2^o$

(b) $(U_1 + U_2)^o = U_1^o \cap U_2^o$

where $U^o \subseteq V^*$ is the annihilator subspace of a vector subspace $U \subseteq V$.

Exercise 2: A bit of algebra. (4 pts)

Let V, W be two finite-dimensional vector spaces, and $\mathrm{Hom}(V, W)$ the vector space of all linear maps from V to W. For $f \in \mathrm{Hom}(V, W)$ define

$$f^* : W^* \to V^*, \qquad \varphi \mapsto \varphi \circ f.$$

(a) Show that f^* is a linear map.

(b) Show that

$$\mathrm{Hom}(V, W) \to \mathrm{Hom}(W^*, V^*), \qquad f \mapsto f^*$$

is a linear map.

Exercise 3: Space duality (4 pts)

Consider the following formulation of the Desargues Theorem as a theorem in $\mathbb{RP}^3$:

> Given two triangles ΔABC and $\Delta A'B'C'$ in $\mathbb{RP}^3$, if joining lines of corresponding vertices meet in a point, then the intersections of corresponding sides lie on a line.

Dualize this theorem in $\mathbb{RP}^3$ (not $\mathbb{RP}^2$!!). Use the letters α, β, γ to represent the dual elements to A, B, C respectively. Attempt to sketch the resulting configuration.

Exercise 4: The complete quadrilateral (4 pts)

Dualize the theorem about complete quadrilateral. Make a sketch of the theorem as well as its dual version and label the corresponding duals as follows: The dual of the point P should be the line p etc. The line spanned by two points P and Q is denoted by PQ.

Due: Before lecture on Wednesday, 30.1.2019.