

Exercise Sheet 13

Exercise 1: Conic sections. (4 pts)

This exercise is concerned with Euclidean, not projective, geometry. Let l be a line in the Euclidean plane and let P be a point not on l . Let e be a positive real number. Consider the set C_e of points X such that the ratio of distances from X to P and to l is equal to e :

$$\text{dist}(X, P)/\text{dist}(X, l) = e.$$

Show that C_e is an ellipse if $e < 1$, a parabola if $e = 1$, and a hyperbola if $e > 1$.

Exercise 2: Singular conics. (4 pts)

Which of the following quadratic forms defines a singular conic? Write those as a product of two linear forms.

(a) $x_0^2 - 2x_0x_1 + 4x_0x_2 - 8x_1^2 + 2x_1x_2 + 3x_2^2$

(b) $x_0^2 - 2x_0x_1 + x_1^2 - 2x_0x_2$

(c) $3x_0^2 - 2x_0x_1 + x_1^2 - 2x_0x_2$

Exercise 3: Conic through five points. (4 pts)

Let $A = [1, 0, 0]$, $B = [0, 1, 0]$, $C = [0, 0, 1]$, $D = [1, 1, 1]$, $E = [2, -1, -1]$ be points in \mathbb{RP}^2 . Find the equation of a conic \mathcal{C} (if it exists) passing through A, B, C, D, E .

Exercise 4: Drawing conics. (4 pts)

Define the quadratic form $q(x) = x_0x_1 + x_1x_2 + x_2x_0$ on \mathbb{R}^3 . Draw a picture of the conic $\mathcal{C} = \left\{ \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \in \mathbb{RP}^2 \mid q(x) = 0 \right\}$ in the (u, v) -plane, where $u = x_0/x_2$ and $v = x_1/x_2$ are affine coordinates on \mathbb{RP}^2 .