TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. Boris Springborn
Hana Kouřimská

Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 2

Exercise 1: Polar triangle. (4 pts)

Suppose Δ is a (small) triangle in S^2 and Δ' is its polar triangle. Show that a point $P \in S^2$ is contained in a Δ if and only if Δ' is contained in the hemisphere H_P centred at P.

Exercise 2: Spherical trigonometry. (4 pts)

Given a spherical triangle with side lengths a, b, c and interior angles α, β, γ, prove the following formula:

$$2 \sin \frac{A}{2} = \frac{\sqrt{\sin s \sin(s-a) \sin(s-b) \sin(s-c)}}{\cos(\frac{a}{2}) \cos(\frac{b}{2}) \cos(\frac{c}{2})},$$

where $A = \alpha + \beta + \gamma - \pi$ is the area of the triangle and $s = \frac{1}{2}(a+b+c)$ is the semiperimeter.

Exercise 3: A special spherical triangle. (5 pts)

Let ΔABC be an isosceles spherical triangle with $d(A, C) = d(B, C)$. (So C lies on the perpendicular bisector of AB.) Let M be the midpoint of AB and let $d(A, M) = d(-M, C)$. Calculate the area of the triangle ΔABC and make a sketch.

Exercise 4: Cosine rule and small triangles. (3 pts)

Prove that for small spherical triangles (with sides $a, b, c < \epsilon << 1$) the spherical law of cosines approximates the Euclidean law of cosines.

Due: Before lecture on Wednesday, 7.11.2018.