TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. Boris Springborn
Hana Kouřimská

Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 8

Exercise 1: Projective space. (5 pts)

Let $P(V)$ be a 3-dimensional projective space. Show that:

(a) Any two planes in $P(V)$ intersect in a line.

(b) Let $P(U_1), P(U_2)$ be two projective subspaces of $P(V)$, defined as projections of linear subspaces U_1, U_2 of V. The projective span of $P(U_1)$ and $P(U_2)$ is defined as

$$P(U_1) + P(U_2) := P(U_1 + U_2).$$

Let k_1, k_2 be the dimensions of $P(U_1), P(U_2)$ resp. Let further k_S be the dimension of $P(U_1) + P(U_2)$ and k_I be the dimension of $P(U_1) \cap P(U_2)$. Show that

$$k_S + k_I = k_1 + k_2.$$

What is the dimension of $P(U_1) \cap P(U_2)$ if the intersection is empty?

Exercise 2: Separating the projective plane with lines. (3 pts)

Into how many regions is the real projective plane separated by three lines, which do not all pass through one point? By four such lines? Optional: by n such lines?

Exercise 3: Projective geometry with different fields. (4 pts)

Let V be the vector space of dimension 3 over the two element field $\mathbb{Z}_2 = \mathbb{Z}/2\mathbb{Z}$. Consider the projective space $P(V)$ with $V = (\mathbb{Z}_2)^3$. How many points does it contain? How many lines? How many points lie on each line? How many lines pass through each point?

Exercise 4: A projective triangle. (4 pts)

Let U_1, U_2 and U_3 be three $2-$dimensional subspaces of $\mathbb{R}^3$ defined by $x = 0$, $x+y+z = 0$ and $3x - 4y + 5z = 0$, respectively. Find the vertices of the triangle in $P(\mathbb{R}^3)$ whose sides are the three projective lines $P(U_i), i \in \{1, 2, 3\}$.

Due: Before lecture on Wednesday, 19.12.2018.