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Geometry I - WS 18

http://www.math.tu-berlin.de/geometrie/Lehre/WS18/GeometryI/

Exercise Sheet 9

Exercise 1: Projective transformations in $\mathbb{R}\mathrm{P}^1$. (6 pts)

Let $Q := \{[0,1],[1,0],[1,2],[2,1]\} \subset \mathbb{R}\mathrm{P}^1$. Consider the projective transformations $\tau : \mathbb{R}\mathrm{P}^1 \to \mathbb{R}\mathrm{P}^1$ such that $\tau(Q) \subseteq Q$.

(a) How many such transformations τ exist?
(b) Find explicit formulas for these transformations τ.
(c) Can a projective transformation $\mathbb{R}\mathrm{P}^1 \to \mathbb{R}\mathrm{P}^1$ which is not the identity have three fixed points?
(d) How many fixed points can a projective transformation $\mathbb{R}\mathrm{P}^1 \to \mathbb{R}\mathrm{P}^1$ have? Give an example for each possible case.

Exercise 4: Skew lines in $\mathbb{R}\mathrm{P}^3$. (3 pts)

(a) Prove that, in general, two lines in $\mathbb{R}\mathrm{P}^3$ do not intersect. Such lines are called *skew* lines.
(b) Given three lines which are pairwise skew, prove that there are infinitely many lines which intersect all three lines.

Exercise 3: Projective transformations in $\mathbb{R}\mathrm{P}^2$. (3 pts)

Given the four points $A_1 = (1,0,1), A_2 = (0,1,1), A_3 = (0,0,1), A_4 = (1,1,1) \in \mathbb{R}^3$ and four more points $B_1 = (1,0,0), B_2 = (0,1,0), B_3 = A_3, B_4 = A_4 \in \mathbb{R}^3$. Let $\hat{f}$ be the projective transformation such that $\hat{f}([A_i]) = [B_i], i \in \{1,2,3,4\}$. Find the element of $PGL(3,\mathbb{R})$ corresponding to $\hat{f}$.

Exercise 4: Desargues' theorem in 3D. (4 pts)

Let P_1, P_2, P_3, P_4 and Q_1, Q_2, Q_3, Q_4 be distinct points in $\mathbb{R}\mathrm{P}^3$ so that neither P_1, P_2, P_3, P_4 nor Q_1, Q_2, Q_3, Q_4 are contained in a plane. Suppose that the four lines P_iQ_i intersect in a common point. Given a pair of corresponding faces, the planes they lie in intersect in a line. Show that all four such lines lie in some common plane.

Additional Christmas Exercise! (1 extra pt)

Simplify the equation such that the text makes sense.

WE WISH YOU A $y = \frac{\ln(\frac{x}{m} - sa)}{r^2}$ AND A HAPPY NEW YEAR!

Due: Before lecture on Wednesday, 9.1.2019.