TECHNISCHE UNIVERSITÄT BERLIN
Faculty II - Institute of Mathematics
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Winter semester 2018/19

Topology: Exercise Sheet 10

(Hurewicz homomorphism, chain homomorphisms)

Exercise 1 (5 points)

Let X, Y, Z be topological spaces and $f : X \to Y$, $g : Y \to Z$ continuous maps.

(a) Let $n \in \mathbb{N}$ be arbitrary. Show that $f_* \circ g_* = (f \circ g)_*$, where $f_* : H_n(X) \to H_n(Y)$ (etc.) are the induced homomorphisms in homology.

(b) Let $h_X : \pi_1(X) \to H_1(X)$ be the Hurewicz homomorphism, and let $\pi_1(f) : \pi_1(X) \to \pi_1(Y)$ denote the induced homomorphism of fundamental groups. Show that $f_* \circ h_X = h_Y \circ \pi_1(f)$.

Exercise 2 (5 points)

Show that every connected, semi-locally simply connected topological space B has a covering $p : X \to B$ such that $H_1(X) = 0$. Does the same hold for higher-dimensional homology groups?

Exercise 3 (5 points)

Show that $\mathbb{R}^m$ is not homeomorphic to $\mathbb{R}^n$ for $n \neq m$.
(Hint: Remove a point from $\mathbb{R}^m$ and $\mathbb{R}^n$ each. You may use material from the lecture on January 21.)

Exercise 4 (5 points)

Find a topological space X with Δ-complex structure such that $H_i(X) \cong H_i(T^2)$ for all $i \in \mathbb{N}$, but X is not homotopy equivalent to $T^2 = S^1 \times S^1$. (We assume the standard Δ-complex structure with two 2-simplices on T^2)

Point total: 20