TECHNISCHE UNIVERSITÄT BERLIN Winter semester 2018/19
Faculty II - Institute of Mathematics
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Topology: Exercise Sheet 12

(Excision, homology manifolds)

Exercise 1 (5 points)

Let A be an index set. For each $\alpha \in A$, let X_α be a topological space and $x_\alpha \in X_\alpha$ be a point such that $(X_\alpha, \{x_\alpha\})$ is a good pair. Calculate the homology groups of the wedge $\bigvee_{\alpha \in A} X_\alpha$ along the x_α.

Exercise 2 (5 points)

A Hausdorff space X is called a *homology manifold of dimension* n if for every $x \in X$, we have that

$$H_i(X, X \setminus \{x\}) = \begin{cases} \mathbb{Z} \text{ if } i = n \\ 0 \text{ otherwise} \end{cases}$$

(a) Let M be an n-dimensional manifold. Show that M is a homology manifold.

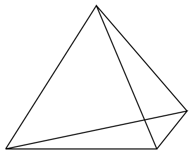
(b) Let X be a topological space such that the cone CX is a homology manifold of dimension n. Show that X has the same homology groups as S^{n-1}.

Exercise 3 (5 points)

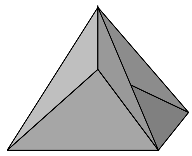
Let $f : X \to Y$ be a continuous map and C_f be the mapping cone of f (see exercise sheet 4). Show that f induces isomorphisms on all homology groups if and only if $\tilde{H}_n(C_f) = 0$ for all n.
(Hint: Consider the cone CX as a subset of C_f.)

Exercise 4 (5 points)

Let X denote the 1-skeleton of the standard 3-simplex Δ^3, that is, X has a Δ-complex structure with 4 vertices and edges connecting each pair of those vertices. Consider the cone CX over X, which can be illustrated by taking the cone point at the barycenter.



X



CX

(a) For each $x \in X$, calculate the homology groups $H_i(X, X \setminus \{x\})$.

(b) For each $x \in CX$, calculate the homology groups $H_i(CX, CX \setminus \{x\})$.

(c) Find as many subspaces $A \subseteq CX$ as possible with the property that each homeomorphism $f : CX \to CX$ maps A into itself (i.e. $f(A) \subseteq A$).

Point total: 20