TECHNISCHE UNIVERSITÄT BERLIN
Faculty II - Institute of Mathematics
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Hand-in date: 16.11.2018

Winter semester 2018/19

Topology: Exercise Sheet 4

(Homotopy equivalence, deformation retraction, degree)

Exercise 1 (5 points)

For a topological space X, define an equivalence relation $\sim$ on X by saying that $x \sim y$ if there is a continuous path from x to y. An equivalence class $[x]$ of $\sim$ is called the *path component* of x.

Show that, if two spaces X and Y are homotopy equivalent, then there is a bijection between the path components of X and the path components of Y.

Exercise 2 (5 points)

Given any continuous map between topological spaces $f : X \to Y$, we can define the *mapping cone* $C_f := ((X \times [0,1]) \sqcup Y)/\sim$, with the equivalence relation generated by $(x, 0) \sim f(x)$ for any $x \in X$ and $(x_1, 1) \sim (x_2, 1)$ for any $x_1, x_2 \in X$.

Now let $f : X \to Y$ and $g : X \to Y$ be two homotopic maps. Show that then, the mapping cones C_f and C_g are homotopy equivalent.

Exercise 3 (5 points)

Consider the 2-dimensional torus $T^2 = S^1 \times S^1$. Show that for any point $p \in T^2$, the space $T^2 \setminus \{p\}$ is homotopy equivalent to $S^1 \vee S^1$, the space consisting of two circles identified at a point.

Exercise 4 (5 points)

Let $f : S^1 \to S^1$ be an *even* map, that is, $f(x) = f(-x)$ for all $x \in S^1$. Show that the degree of f is even.

Point total: 20