TECHNISCHE UNIVERSITÄT BERLIN
Faculty II - Institute of Mathematics
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Winter semester 2018/19

Topology: Exercise Sheet 9

(Δ-complexes, simplicial and singular homology)

Exercise 1 (5 points)

We can obtain a space with Δ-complex structure homeomorphic to S^3 by gluing two 3-simplices along their boundary (i.e. $X = (\Delta^3 \times \{0,1\})/\sim$, where $(x,0) \sim (x,1)$ iff $x \in \partial\Delta^3$). Use this structure to calculate the homology groups of S^3.

Exercise 2 (5 points)

Let X, Y be topological spaces, and $f : X \to Y$ a continuous map. We can define homomorphisms $f_q : C_q(X) \to C_q(Y)$ by setting, for each q-dimensional simplex $\sigma : \Delta^q \to X$, $f_q(\sigma) := f \circ \sigma$.

(a) Show that $\partial \circ f_q = f_{q-1} \circ \partial$ (we say that (f_q) is a *chain homomorphism* in this case).

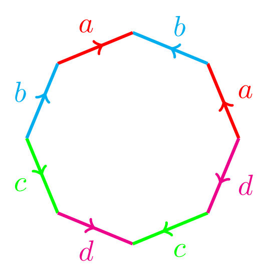
(b) Use this to obtain induced homomorphisms $f_* : H_q(X) \to H_q(Y)$.

Exercise 3 (5 points)

For any n, find a Δ-complex structure on $\mathbb{R}P^n$.

Exercise 4 (5 points)

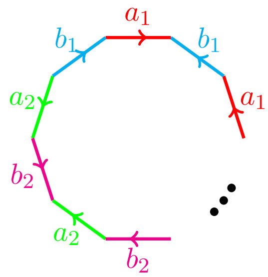
Consider the surface F_2 of genus 2, obtained by gluing an octagon along its boundary in the way indicated in the picture. Compute the homology groups of F_2.



Exercise 5* (5 points)

Consider, for any $n \geq 1$, the surface F_n of genus n, obtained by gluing a $4n$-gon along its boundary in the way indicated in the picture. Compute the homology groups of F_n.

(This exercise is voluntary and worth extra points.)



Point total: 20 (+5)