Technische Universität Berlin
Institut für Mathematik
Knöppel
http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/CAII/ WS 19

# Complex Analysis II: Riemann Surfaces

## Exercise Sheet 1

(Holomorphic maps)

due 30.10.2019

**Exercise 1** **2 points**

Let $U \subset \mathbb{C}^m$. Show that a map $f\colon U \to \mathbb{C}^n$ is holomorphic if and only if for $i = 1, \ldots, n$ the component functions $f_i\colon U \to \mathbb{C}$ are holomorphic.

**Exercise 2** **2 points**

For $i = 1, 2$ let $A_i$ denote the annulus around zero given by the radii $0 < r_i < R_i$,

$$A_i := \{z \in \mathbb{C} \mid r_i < |z| < R_i\}.$$

Show that $A_1$ and $A_2$ are diffeomorhic. Show furthermore that $A_1$ and $A_2$ are biholomorphic if and only if $R_1/r_1 = R_2/r_2$.

**Exercise 3** **2 points**

Let $f$ be holomorphic on a closed polydisk $D_{a,r} \subset \mathbb{C}^m$ and $|f(z)| < M$ for all $z \in D_{a,r}$. Show that, for $k \in \mathbb{N}^m$,

$$|f^{(k)}(a)| < \frac{k!M}{r^k}\,.$$