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http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/CAII/ WS 19

Complex Analysis II: Riemann Surfaces

Exercise Sheet 2

(Manifolds)

due 8.11.2019

Exercise 1 **2 points**

Let M be a manifold. Show that M is connected if and only if M is path-connected.

Exercise 2 **2 points**

Let $\mathbb{CP}^n$ denote the n-dimensional complex projective space (cf. Example 2.1.13). For $j = 1, \ldots, n+1$, let $U_j := \{[z] \in \mathbb{CP}^n \mid z_j \neq 0\}$ and

$$\varphi_j \colon U_j \to \mathbb{C}^n\,, \quad [(z_1, \ldots, z_{n+1})] \mapsto (z_1, \ldots, \widehat{z_j}, \ldots, z_{n+1})/z_j\,.$$

Show that $\{(U_j, \phi_j)\}_{j=1,\ldots,n+1}$ is a complex atlas of $\mathbb{CP}^n$.

Exercise 3 **2 points**

Show that the Riemann sphere $\widehat{\mathbb{C}}$ (cf. Example 2.1.14) is biholomorphic to the 1-dimensional complex projective space $\mathbb{CP}^1$. Furthermore, show that biholomorphisms $\widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ are exactly the Möbius transformations.