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http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/CAII/ WS 19

Complex Analysis II: Riemann Surfaces

Exercise Sheet 3

(Submanifolds)

due 15.11.2019

Exercise 1 **2 points**

Let M, $\tilde{M}$ be two 1-dimensional complex submanifolds of $\mathbb{C}^2$. Show that if they intersect, then they intersect either in an isolated point or M and $\tilde{M}$ locally coincide.

Exercise 2 **2 points**

Let $\mathbb{S}^{2n+1} \subset \mathbb{R}^{2n+2} = \mathbb{C}^{n+1}$ denote the unit sphere and let $\pi \colon \mathbb{S}^{2n+1} \to \mathbb{CP}^n$ denote the restriction of the canonical projection $(\mathbb{C}^{n+1})^\times \to \mathbb{CP}^n$. Show that π is a smooth submersion all of whose fibers are diffeomorphic to $\mathbb{S}^1$.

Exercise 3 **2 points**

The Veronese map is a map $\vartheta_d \colon \mathbb{CP}^n \to \mathbb{CP}^m$, where $m = \binom{n+d}{d} - 1$, which sends $[(z_0, \dots, z_n)] \in \mathbb{CP}^n$ to the complex line spanned by monomials in $z_0, \dots, z_n$ of degree d. For $n = 1$ and $d = 3$, we have

$$\vartheta_3 \colon \mathbb{CP}^1 \to \mathbb{CP}^3, \;\; [(z, w)] \mapsto [(z^3, z^2 w, z w^2, w^3)] \,.$$

Show that

(a) ϑ_3 is well-defined and holomorphic,

(b) $\vartheta_3(\mathbb{CP}^1)$ is a compact submanifold of $\mathbb{CP}^3$.