Technische Universität Berlin
Institut für Mathematik
Knöppel
http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/CAII/ WS 19

Complex Analysis II: Riemann Surfaces

Exercise Sheet 6

(Connections)

due 06.12.2019

Exercise 1 **2 points**

Let $E \to M$ be a vector bundle of rank r with connection ∇ and σ be a parallel frame. Then

$$\nabla(\sigma.\nu) = \sigma.(d\nu), \quad \forall\ \nu \in \mathscr{C}^\infty(M;\mathbb{R}^r)\,.$$

Exercise 2 **2 points**

Let $E \to M$ be a vector bundle with connection ∇. Then for each $p \in M$ there exists a local frame $\psi_1, \ldots, \psi_r \in \Gamma E$ at p such that

$$\nabla\psi_1|_{T_pM} = \cdots = \nabla\psi_r|_{T_pM} = 0\,.$$

Exercise 3 **2 points**

Let $E = E_1 \oplus E_2$ be a vector bundle with connection ∇, then there are connections ∇^i on E_i, $A \in \Omega^1(M;\mathrm{Hom}(E_1;E_2))$ and $B \in \Omega^1(M;\mathrm{Hom}(E_2;E_1))$ such that

$$\nabla = \begin{pmatrix} \nabla^1 & B \\ A & \nabla^2 \end{pmatrix}\,.$$