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http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/CAII/ WS 19

Complex Analysis II: Riemann Surfaces

Exercise Sheet 9

(Stokes' theorem, Poincaré–Hopf index theorem)

due 10.01.2020

Exercise 1 **2 points**

Let M be a compact oriented Riemannian manifold with boundary ∂M. The divergence $\operatorname{div}\colon \Gamma TM \to \mathscr{C}^\infty M$ is defined by $\operatorname{div}(X)\, d\mathrm{vol}_M := d(X \lrcorner\, d\mathrm{vol}_M)$. Show:

$$\int_M \operatorname{div}(X) = \int_{\partial M} \langle X, N \rangle \,,$$

where N denotes the outward-pointing unit normal field along ∂M.

Exercise 2 **2 points**

Let $E \to M$ be a complex line bundle over an oriented surface and $\psi \in \Gamma E$ transverse to the zero section. Show that the zeros of ψ have index ± 1.

Exercise 3 **2 points**

The tangent bundle of $\mathbb{S}^2 \subset \mathbb{R}^3$ is a complex line bundle with almost complex structure $J_p\colon T_p\mathbb{S}^2 \to \mathbb{S}^2$ given by $J_pX = p \times X$. Compute the degree of $T\mathbb{S}^2$.