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http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/CAII/ WS 19

# Complex Analysis II: Riemann Surfaces

## Exercise Sheet 12

($\bar\partial$-problems)

due 31.01.2020

**Exercise 1** **2 points**

Let $E$ be a complex vector bundle with connection $\nabla$ over a compact almost complex surface $M$ and let $\omega \in \Omega^2(M;E)$. Show:

$$\exists\, \eta \in \Gamma KE \colon \omega = d^\nabla \eta \iff \forall\, \varphi \in H^0E^* \colon \langle\!\langle \varphi | \omega \rangle\!\rangle = 0\,.$$

**Exercise 2** **2 points**

Let $E$ be a complex vector bundle with connection $\nabla$ over a compact almost complex surface $M$ and let $\eta \in \Gamma \bar{K} E$. Show:

$$\exists\, \psi \in \Gamma E \colon \eta = \bar\partial \psi \iff \forall\, \varphi \in H^0 K E^* \colon \langle\!\langle \varphi | \eta \rangle\!\rangle = 0\,.$$

**Exercise 3** **2 points**

Let $M$ be a compact almost complex surface and let $\alpha \in \Omega^1 M$. Show that

$$\alpha \ \textit{exact} \iff \int_M \alpha \wedge \beta = 0 \ \textit{ for all closed } \beta \in \Omega^1 M\,.$$