TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Geometry I - WS 2019/2020

http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/GeometryI/

Exercise Sheet 1

Exercise 1: Intersections of projective subspaces. (2 pts)

(i) Two distinct lines in a projective plane always intersect in a point

(ii) Two distinct planes in a 3-dimensional projective space always intersect in a line.

Exercise 2: Skew projective lines. (2 pts)

(i) Prove that two lines in $\mathbb{R}\mathrm{P}^3$ do not intersect unless they lie in a common projective plane. Such lines are called *skew* lines.

(ii) Given three lines which are pair-wise skew, prove that there are an infinite number of lines which intersect all three lines.

Exercise 3: Decomposition of the projective plane by lines. (2 pts)

Into how many regions is the real projective plane separated by n lines in general position, i.e., n lines such that no three pass through one point?

Exercise 4: A small projective plane. (2 pts)

Let V be the vector space of dimension 3 over the two element field $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$. Consider the projective space $\mathrm{P}(V)$ with $V = (\mathbb{F}_2)^3$. How many points does it contain? How many lines? How many points lie on each line? How many lines pass through each point? Draw a schematic picture of the configuration.

Due Thursday, October 31.