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Geometry I - WS 2019/2020

http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/GeometryI/

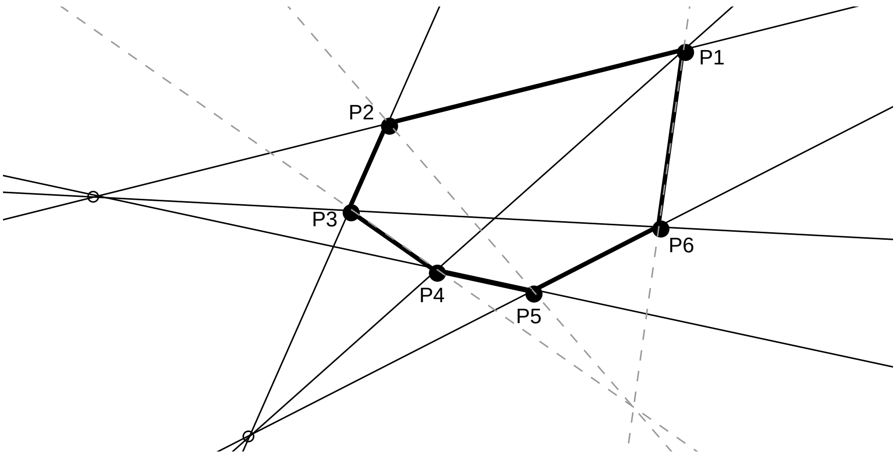
Exercise Sheet 2

Exercise 1: Three parallel lines. (2 pts)

Let A, B, A', B' be four points in the plane such that the lines (AA') and (BB') are parallel. Let $\tilde{C} := (AB) \cap (A'B')$ and $\tilde{A}, \tilde{B}$ be two points on a line through $\tilde{C}$. Let $C := (\tilde{A}B) \cap (A\tilde{B})$, $C' := (\tilde{A}B') \cap (A'\tilde{B})$. Show that the line (CC') is parallel to (AA').

Exercise 2: An incidence theorem. (2 pts)

Let $P_1, P_2, P_3, P_4, P_5, P_6$ be distinct points in the projective plane $\mathbb{R}\mathrm{P}^2$. Suppose that the three lines P_1P_2, P_4P_5, P_3P_6, as well as the three lines P_2P_3, P_5P_6, P_4P_1 intersect in one point. Show that the lines P_3P_4, P_6P_1, P_5P_2 also intersect in one point.



Exercise 3: Affine transformations. (2 pts)

Let $f : \mathbb{R}\mathrm{P}^2 \to \mathbb{R}\mathrm{P}^2$ be a projective map that sends the line at infinity $\{[x_0, x_1, x_2] \in \mathbb{R}\mathrm{P}^2 \,|\, x_0 = 0\}$ to the line at infinity. Show that in affine coordinates $y_i = \frac{x_i}{x_0}$ a point $y = \left(\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right)$ is mapped to $Ay + b$ for some $A \in GL(2, \mathbb{R})$, $b \in \mathbb{R}^2$.

Exercise 4: Three-dimensional version of Desargues. (2 pts)

Consider two tetrahedra in $\mathbb{R}\mathrm{P}^3$ with vertices P_1, P_2, P_3, P_4 and Q_1, Q_2, Q_3, Q_4, respectively, that are in perspective with respect to a point S (i.e. the lines (P_iQ_i), $i = 1, \ldots, 4$ all go through the point S). Show that the lines of intersection $l_i := (P_jP_kP_l) \cap (Q_jQ_kQ_l)$ $(i, j, k, l = 1, \ldots, 4$ distinct$)$ of corresponding faces of the tetrahedra lie in a plane. State a suitable assumption of genericity for the vertices of the tetrahedra.