TECHNISCHE UNIVERSITÄT BERLIN
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Geometry I - WS 2019/2020

http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/GeometryI/

Exercise Sheet 4

Exercise 1: Multiratio of a polygon. (2 pts)

Let $A_1, \dots, A_k$ be distinct points in $\mathbb{RP}^n = \mathbb{R}^n \cup \mathbb{RP}^{n-1}$. Let $B_i \in (A_i A_{i+1})$ be points on the lines $(A_i A_{i+1})$, $i = 1, \dots, k$ with cyclic indices. Show that the quantity

$$m := \frac{\ell(A_1 B_1)}{\ell(B_1 A_2)} \cdot \ldots \cdot \frac{\ell(A_k B_k)}{\ell(B_k A_1)},$$

where $\ell(PQ)$ denotes the directed length from P to Q, is a projective invariant.

Exercise 2: Dualization. (2 pts)

Dualize the following construction by writing out the dual of each step, and provide a legible, labeled drawing of the construction and its dual, and prove the claim.
Note: The dual of the point P should be the line p, etc.

1. On a given line l, choose three points P, Q, and X.
2. Choose line p passing through P, q passing through Q, and x passing through X such that the three lines do not pass through one point.
3. $A := pq$, $B := px$, $C := qx$
4. $s := PC$, and $r := QB$
5. $D := sr$
6. $y := AD$
7. $Y := yl$. Then $cr(Y, P, X, Q) = -1$.

Exercise 3: Euclidean conic sections. (2 pts)

This exercise is concerned not with projective, but with Euclidean geometry. Let l be a line in the Euclidean plane and let P be a point not on l. Let e be a positive real number. Consider the set C_e of points X such that the ratio of distances from X to P and to l is equal to e:

$$\frac{\mathrm{dist}(X, P)}{\mathrm{dist}(X, l)} = e.$$

Show that C_e is an ellipse if $e < 1$, a parabola if $e = 1$, and a hyperbola if $e > 1$.

Exercise 4: Affine conic sections. (2 pts)

Consider the intersections of an ellipse in the affine plane $\mathbb{R}^2$ with a family of parallel lines. This gives a family of line segments contained in the interior of the ellipse. Show that the midpoints of these line segments are collinear.

Due Thursday, November 21.