TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Mathematical Physics I, Dr. Matteo Petrera, René Zander

WS 19/20

http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/MP1/

Exercise Sheet 7 (03.12.19)

Due date: **10.12.19**

- To get the Übungsschein (necessary condition for the oral exam) you need to collect *60% of the total sum of points in each half of the semester*. Each exercise sheet has 20 points. The total number of sheets is 12-14.
- Please work in fixed groups of *2 or 3 students*.
- Please justify each step of your computations. *Results without any explanation are not accepted*. Please write in a readable way. *Unreadable handwriting will not be corrected*. Feel free to write your answers either in English or in German.
- Please turn in your homework directly to me at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed*.

Exercise 1 (4 pts)

Consider the following IVP in $\mathbb{R}^2$:

$$\begin{cases} \dot{x}_1 = -x_1, \\ \dot{x}_2 = x_2 + x_1^2, \\ (x_1(0), x_2(0)) \in \mathbb{R}^2, \end{cases}$$

1. Determine $\alpha \in \mathbb{R}$ such that

$$\Phi_t(x_1(0), x_2(0)) = \left(x_1(0)e^{-t}, x_2(0)e^t + \alpha x_1^2(0)(e^t - e^{-2t})\right)$$

   is the flow of the ODE.

2. Linearize the system around the fixed point $(0,0)$ and find the stable and unstable linear subspaces.

3. Construct the stable and unstable manifolds of the nonlinear system.

Exercise 2 (4 pts)

Consider the following non-autonomous IVP in $\mathbb{R}^2$:

$$\begin{cases} \dot{x}_1 = x_1 + x_2, \\ \dot{x}_2 = a(t)x_2, \\ (x_1(0), x_2(0)) \in \mathbb{R}^2, \end{cases}$$

where

$$a(t) = \frac{\cos t + \sin t}{2 + \sin t - \cos t}.$$

1. Find the principal matrix solution.

2. Compute the Floquet multipliers. What can you say about the stability of $(0,0)$?

TURN OVER!

Exercise 3 (4 pts)

Consider the scalar ODE:

$$\dot{x} + \alpha x - \sin x, \quad \alpha \geq 0.$$

- Fix $\alpha = 0$. Find the fixed points, study their stability and sketch the phase portrait.
- Fix $\alpha > 1$. Show that there is only one fixed point. Is it stable?
- Find and classify the bifurcations that occur as α is varied and $-\pi < x < \pi$.

Exercise 4 (4+4 pts)

Consider the following two scalar ODEs:

1. $\dot{x} = x - \alpha\, x(1 - x)$,
2. $\dot{x} = x + \dfrac{\alpha\, x}{1 + x^2}$.

For both of them:

- Find and classify the bifurcations that occur as α is varied.
- Draw the bifurcation diagram.