TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Mathematical Physics I, Dr. Matteo Petrera, René Zander — WS 19/20

http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/MP1/

Exercise Sheet 10 (07.01.20)

Due date: **14.01.20**

- To get the Übungsschein (necessary condition for the oral exam) you need to collect *60% of the total sum of points in each half of the semester*. Each exercise sheet has 20 points. The total number of sheets is 12-14.
- Please work in fixed groups of *2 or 3 students*.
- Please justify each step of your computations. *Results without any explanation are not accepted*. Please write in a readable way. *Unreadable handwriting will not be corrected*. Feel free to write your answers either in English or in German.
- Please turn in your homework directly to me at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed*.

Exercise 1 (6 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^2$ consider the the parametric family of vector fields

$$f(q,p) := \left(p^\alpha q^\beta, -p^{\alpha+1} q^\delta\right),$$

with $\alpha, \beta, \delta \in \mathbb{R}$.

1. Find the values of α, β, δ for which the f is Hamiltonian.
2. Compute the corresponding Hamiltonians.

Exercise 2 (4 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^{2n}$ consider the transformation

$$(q,p) \mapsto (\widetilde{q}, \widetilde{p}) := (q, f(q,p)),$$

for some smooth function f. Determine the structure that f must have for the transformation to be symplectic.

Exercise 3 (4 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^2$ consider the transformation

$$(q,p) \mapsto (\widetilde{q}, \widetilde{p}) := \left(q\sqrt{1+q^2p^2}, \frac{p}{\sqrt{1+q^2p^2}}\right).$$

Show that this transformation is symplectic.

Turn over

Exercise 4 (6 pts)

In the canonical Hamiltonian phase space $\mathbb{R}^4$ consider the Hamiltonian

$$\mathcal{H}(q_1, q_2, p_1, p_2) := \frac{1}{2}\left(p_1^2 + q_1^2\, q_2\, p_2\right).$$

1. Find a one-parameter group of symplectic transformations

$$(q_1, q_2, p_1, p_2) \mapsto (\widetilde{q}_1, \widetilde{q}_2, \widetilde{p}_1, \widetilde{p}_2) = \Psi_s(q_1, q_2, p_1, p_2), \qquad s \in \mathbb{R},$$

   which preserves the form of the function $\mathcal{H}$ for all $s \in \mathbb{R}$, i.e.,

$$\mathcal{H}\left(\widetilde{q}_1, \widetilde{q}_2, \widetilde{p}_1, \widetilde{p}_2\right) = \frac{1}{2}\left(p_1^2 + q_1^2\, q_2\, p_2\right).$$

   Hint: split the variables in pairs (q_1, p_1) and (q_2, p_2). You can choose transformations that act as the identity on one of these pairs.

2. Construct the infinitesimal generator $\mathbf{v}$ of Ψ_s.

3. Find a Hamiltonian of the vector field $\mathbf{v}$ and verify that this new Hamiltonian is an integral of motion of the flow of the original Hamiltonian $\mathcal{H}$.