Proposition: A non-degenerate quadric $Q \subset \mathbb{R}^3 \subset \mathbb{RP}^3$ is a sphere, if and only if it contains the absolute imaginary conic at infinity (upon complexification):

$$\mathcal{Z}: \quad x_1^2 + x_2^2 + x_3^2 = 0 \quad , \quad x_4 = 0$$

Proof:

"$\Rightarrow$" Q is a sphere

apply similarity transformation $\rightsquigarrow$ $Q = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 = 0$$

for $x_4 = 0$: $x_1^2 + x_2^2 + x_3^2 = 0$

"$\Leftarrow$" Q contains $\mathcal{Z}$

thus Q cannot be tangent to the plane at infinity

apply Euclidean transformation $\rightsquigarrow$ $Q = \begin{pmatrix} a & & & \\ & b & & \\ & & c & \\ & & & -1 \end{pmatrix}$

$$a x_1^2 + b x_2^2 + c x_3^2 - x_4^2 = 0$$

for $x_4 = 0$: $a x_1^2 + b x_2^2 + c x_3^2 = 0$,

which coincides $\mathcal{Z}$ if and only if $a = b = c$.