A structural-algebraic approach for coupled DAE systems

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Modeling and simulation of multi-physical systems is an important issue in many industrial applications. In modern simulation packages such as DYMOLA or MATLAB/SIMULINK the modeling of multi-physical systems is usually based on a network of subsystems for the different physical domains coupled together via certain interface conditions. This modeling of dynamical systems via interconnected subsystems leads to differential-algebraic equations (DAEs) that require certain regularization techniques to guarantee stable and robust numerical computations [1]. In most simulation environments computer-algebra packages and symbolic differentiation are used to identify the constraints and interface conditions based on generic structural information and to resolve these conditions in order to obtain a system in minimal coordinates. However, even for very simple examples an index reduction based on structural properties, as e.g. used by Pantelides algorithm [3] or in the structural analysis of Pryce [4], can fail if the system is *structurally singular*.

In this contribution a new remodeling approach for coupled DAE systems is introduced that combines the structural information of the coupling network with more sophisticated index reduction techniques. We consider coupled systems of DAEs where each subsystem $\mathcal{S}_i$ is given by a semi-explicit d-index-1 system of the form

$$\dot{x}^i = f^i(x^i, y^i, u^i), \tag{1}$$

$$0 = g^i(x^i, y^i, u^i), \tag{2}$$

with differential state variables x^i, algebraic state variables y^i and inputs u^i, and nonsingular Jacobian $\frac{\partial g^i}{\partial y^i}$. For each uni-physical subsystem usually such a regularized d-index-1 formulation can be easily obtained based on structural information. The coupling between two subsystems $\mathcal{S}_i$ and $\mathcal{S}_j$ is done via coupling conditions of the form

$$u^i = G_{i,j}(x^j, y^j),$$

describing the connection of the output of subsystems $\mathcal{S}_j$ to the input of subsystem $\mathcal{S}_i$. Even if both subsystems $\mathcal{S}_i$ and $\mathcal{S}_j$ are d-index-1 systems, a cyclic coupling can easily lead to a high index DAE. We propose a new remodeling approach that basically consists of two steps. At first the coupling structure is analyzed and configurations that can lead to an increase in the index are identified. We will present conditions that specify in which cases the coupled system is again of d-index 1, and in which cases an increase of the index due to coupling occurs. If this is the case, then an index reduction technique based on the idea of minimal extension [2] is used in the second step to produce a slightly extended d-index-1 system using the system equations and some of its derivatives and introducing a small number of new variables. The presented results are still work in progress.

References

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