Space-time adaptive discontinuous Galerkin methods for advection dominated reactive flows

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Diffusion-convection-reaction equations are widely used for systems in chemical reaction problems. In the linear convection dominated case, stabilized continuous finite elements and discontinuous Galerkin (DG) methods are capable of handling the nonphysical oscillations. On the other hand, in the nonlinear stationary case, the nonlinear reaction terms produces sharp layers in addition to the spurious oscillations due to the convection. Thus, an accurate and efficient numerical resolution of such layers is a challenge as the exact location of the layers are not known a priori. In the non-stationary case, the resolution of such layers is more critical since the nature of the sharp layers may vary as time progresses. In contrast to the stabilized continuous Galerkin finite element methods [1], DG methods produce stable discretizations without the need for stabilization parameters. Moreover, DG methods are better suited for adaptive strategies.

In this talk, we apply the time-space adaptive algorithm in [2], which is based on utilizing the elliptic reconstruction technique to be able to use the robust (in Péclet number) elliptic error estimator in [3], for the convection dominated problems with nonlinear reaction mechanisms. We derive a posteriori error estimates in the $L^2(H^1)$ and $L^\infty(L^2)$-type norms using backward Euler in time and discontinuous Galerkin (symmetric interior penalty Galerkin, SIPG) in space. Numerical results for reactive flows with polynomial and Arrhenius type nonlinearity and in heterogeneous porous media [4] will be presented.

References

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